

Algorithm:

Merge Step:

Merge two sorted lists into one sorted list in linear time.

Let Q1 and Q2 be two sorted queues.

Let Q be an empty queue.

```
// One of these queues will eventually empty out and
```

```
// break from loop
```

```
while (Q1 and Q2 are not empty) {
```

```
item1 = Q1.front();
```

```
item2 = Q2.front();
```

Add $\min(\text{item1}, \text{item2})$ from (Q1 or Q2) to end of Q

}

concatenate non-empty(Q1, Q2) to end of Q

Actual Algorithm:

Start with unsorted list l of n items

Break into two lists with middle index (I1, I2)

Sort I1 recursively, yielding S1

Sort I2 recursively, yielding S2

Merge S1 and S2 into one sorted list S

Properties:

- $O(n \log(n))$ time
- $O(n)$ memory required for arrays (merge step)
 - $O(\log(n))$ memory for recursion stack space but merge step memory dominates the stack space
- $O(\log(n))$ memory for linked list (merge can be done in $O(1)$)
- Recursive divide-and-conquer algorithm
 - Dividing simple
 - Do all working merging step
- Natural for Linked Lists (In-Place)
 - QuickSort requires pivot, this does not.
 - MergeSort doesn't require random access like QuickSort does.
- Not In-Place Sort For Arrays
- Stable Sorting Algorithm
- Great if numbers larger than memory
 - External Merge Sort
- Worse Locality of Reference Than QuickSort

```
public static void mergeSort(int arr[]) {
    mergeSort(arr, new int[arr.length], 0, arr.length - 1);
}

/** The fullCopy variable is the O(n) memory */
private static void mergeSort(int arr[], int fullCopy[], int low, int high) {
    if (low < high) {
        int mid = (low + high) / 2;
        // Sort Left Half Infinitely [low, mid]
        mergeSort(arr, fullCopy, low, mid);
        // Sort Right Half Infinitely [mid + 1, high]
        mergeSort(arr, fullCopy, mid + 1, high);
        // Merge Two Sorted Halves
        merge(arr, fullCopy, low, mid, high);
    }
}
```

```
private static void merge(int arr[], int fullCopy[], int low, int mid, int high) {
    // Copy Array From [low, high] into fullCopy[]
    System.arraycopy(arr, low, fullCopy, low, high - low + 1);
```

```
int leftPtr = low;
```

```
int rightPtr = mid + 1;
```

```
int current = low;
```

```
// Grabbing smallest from left and right halves of the
// array and putting them into the original array
```

```
while(leftPtr <= mid && rightPtr <= high){
```

```
if(fullCopy[leftPtr] <= fullCopy[rightPtr]){
```

```
arr[current] = fullCopy[leftPtr];
```

```
leftPtr++;
```

```
} else {
```

```
arr[current] = fullCopy[rightPtr];
```

```
rightPtr++
```

}

```
current++;
```

```
// Finishing off remaining left over array parts
```

```
while(leftPtr <= mid){
```

```
arr[current++] = fullCopy[leftPtr++];
```

}

```
while(rightPtr <= high){
```

```
arr[current++] = fullCopy[rightPtr++];
```

}

1



The diagram illustrates the Merge Sort algorithm. It shows an array [7, 3, 9, 5, 4, 8, 0, 1] being recursively split into smaller sub-arrays until single elements are reached. The sub-arrays are then merged back together in sorted order. The diagram highlights the merging process at each level, showing how two sorted sub-arrays are combined into a larger sorted array. The final result is a fully sorted array [0, 1, 3, 4, 5, 7, 8, 9].

Start Merge Sort

0 1 3 4 5 7 8 9

Compare Left and Right to Get Top

0 1 4 8

Compare Left and Right to Get Top

0 1

1 + $\lceil \log 2(n) \rceil$ levels

O(n) Work Per Level: Merging Lists Together On Each Level

Therefore O(n log(n)) Running Time



Merge Algorithm

Merge Algorithm				Current	Left Ptr	Right Ptr	
3 7 2 8							
1. Copy Array Into FullCopy							
Original Array	3	7	2	8	0	0	2
Full Copy Array	3	7	2	8			
	Left Pointer		Right Pointer				
Original Array	2	7	2	8	1	1	2
Full Copy Array	3	7	2	8			
	Left Pointer		Right Pointer				
Original Array	2	3	2	8	2	1	3
Full Copy Array	3	7	2	8			
	Left Pointer		Right Pointer				
Original Array	2	3	7	8	3	2	3
		Left Pointer	Right Pointer				
Wait! Left pointer is out of bounds. We now just copy what is left from rightPtr to end of array. Coincidence, it is already sorted.							